PROPOSITIONAL LOGIC

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LOGIC

- Logic is concerned with the reasoning and the validity of arguments.
- Based on Truth Values (True, False)
- A representational method used in AI

- Formal system to describe states of affairs
  - Syntax
  - Semantics

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Syntax

- How the symbols in a language can be put together in a correct way
- Does NOT imply anything about the correctness about their interpretation (3 = 2 is syntactically correct)

- Can be expressed in a formal way

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PROPOSITIONAL LOGIC

- Symbols:
  - True
  - False
  - Proposition symbols
  - Logical connectives
  - Parenthesis ()

- Sentences
  - True & False are sentences
  - Propositional symbol, e.g., is a sentence by itself
  - Wrapping parenthesis around a sentence yields a sentence, e.g., $(P \wedge Q)$
  - Can be formed from simpler sentences by using connectives

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CONNECTIVES

- $\wedge$: Conjunction (parts are conjuncts)
- $\vee$: Disjunction (parts are disjuncts)
- $\Rightarrow$: Implication: $(P \wedge Q) \Rightarrow R$. Premise or antecedent is $(P \wedge Q)$ and consequent is R. Implications are also called rules or if-then statements.
- $\neg$: Not. The sentence $\neg$ P is called negation of P
- $\Leftrightarrow$: Double Implication !!

SENTENCES IN PROPOSITIONAL LOGIC

- Sentence → Atomic sentence | Complex sentence
- Atomic sentence → True | False | Symbol
- Symbol → P | Q | R| …
- Complex sentence → (Sentence)

  | Sentence (some connective) sentence

  | $\neg$ Sentence
- Connective → $\vee$ | $\wedge$ | $\Leftrightarrow$ | $\Rightarrow$

Sentences

- If P and Q are sentences, then
  - $\neg P$
  - $P \vee Q$
  - $P \wedge Q$
  - $P \Rightarrow Q$
  - Q =false
  - (Q)
- are also sentences

Ambiguity in Grammar

- Our grammar as defined is ambiguous
- Resolve this with operator precedence
- highest $\neg \mid \wedge \mid \vee \mid \Rightarrow \mid =$ lowest
- ex: $\neg P \wedge Q \vee R \Rightarrow S$ is in fact $((\neg P) \vee (Q \wedge R)) \Rightarrow S$
- Use of brackets to remove ambiguity

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Semantics

- Defines our interpretation of symbols and expressions.

- Symbols means whatever facts you like
  - P could mean “Sky is blue”
  - Q could mean “Elephant is an animal”
  - Truth symbols (often T and F) are obvious

- Note that is generally much harder to formally define semantics than syntax.

MODEL

- Any world in which a sentence is true under a particular interpretation is called a model
- Models are mathematical objects
- A model fixes truth values for every propositional symbol
  - Truth values are true or false
  - True: The fact holds (the way the world is)
  - False: The fact does not hold (the way the world is not)
- Truth tables
  - Describe output of logical functions
  - Test for valid sentences
- Define truth tables for each symbol

Truth table

P	Q	$P \Leftrightarrow Q$
f	f	t
t	f	f
f	t	f
t	t	t

P	Q	$P \Rightarrow Q$
f	f	t
t	f	f
f	t	t
t	t	t

We have 4 models/possible worlds

Identities

double negation	$\neg(\neg P) = P$
unit resolution	$(P \vee Q) = (\neg P \Rightarrow Q)$
contraposition	$(P \Rightarrow Q) = (\neg Q \Rightarrow \neg P)$
De Morgan	$\neg(P \wedge Q) = (\neg P \vee \neg Q)$
	$\neg(P \vee Q) = (\neg P \wedge \neg Q)$

RULES OF INFERENCE

- Inference rules provide a computationally feasible way to determine when an expression logically follows from another.

- Modus Ponens
  $\alpha \Rightarrow \beta, \alpha \vdash \beta$

- Modus Tolens
  $\alpha \Rightarrow \beta, \neg \beta \vdash \neg \alpha$

- And-Elimination
  $\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n \vdash \alpha_i$

- And Introduction
  $\alpha_1, \alpha_2, \ldots, \alpha_n \vdash \alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n$

RULES OF INFERENCE

- Or-introduction
  $\alpha_i \vdash \alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_n$
- Double negation elimination
  $\neg \neg \alpha \vdash \alpha$
- Unit resolution
  $\alpha \vee \beta, \neg \beta \vdash \alpha$
- Resolution: Very important rule of inference
  $\alpha \vee \beta, \neg \beta \vee \gamma \vdash \alpha \vee \gamma$ (resolvent)
  $\neg \alpha \Rightarrow \beta, \beta \Rightarrow \gamma \vdash \neg \alpha \Rightarrow \gamma$

SOUNDNESS / COMPLETENESS

- Sound: correct
- Complete: No other rule required